Michal Šprlák

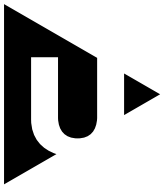
PHYSICAL GEODESY



FACULTY OF APPLIED SCIENCES
UNIVERSITY OF WEST BOHEMIA
IN PILSEN

Michal Šprlák

PHYSICAL GEODESY



FACULTY OF APPLIED SCIENCES
UNIVERSITY OF WEST BOHEMIA
IN PILSEN

PHYSICAL GEODESY

Michal Šprlák
Department of Geomatics, Faculty of Applied Sciences, University of West Bohemia in Pilsen

Reviewer: prof. Ing. Jan Kostelecký, DrSc.
Cover page: Tereza Saitzová

Published by:
University of West Bohemia in Pilsen, Univerzitní 2732/8, 301 00 Pilsen, Czech Republic

1st Edition, 197 pages
Pilsen 2026

ISBN 978-80-261-1401-7

DOI https://doi.org/10.24132/ZCU.2026.14017

© University of West Bohemia in Pilsen, 2026

Preface to the Czech version

Gravitation is a natural phenomenon reflecting the state of mutually attracting masses and is one of the four fundamental physical interactions. Knowledge of it is essential, for example, in: 1) realisations of planetary reference systems for positioning and predicting the orbits of artificial satellites in geodesy; 2) the study of internal composition, subsurface processes, and the thermal evolution of planetary bodies in geophysics and planetary sciences; 3) monitoring mass transport to understand climate change in environmental sciences; 4) the navigation of ground and space vehicles in transportation, space exploration, and military operations. In general, gravitation helps in scientific, industrial, and research advancements, as well as in solving a wide range of societal issues, such as sustainable energy, environmental aspects, or infrastructure development.

The discipline concerned with determining macroscopic gravitational fields is called physical geodesy. It is an integral part of the International Association of Geodesy and is officially considered one of the three main pillars of modern geodetic research. The prominent position of physical geodesy is also demonstrated by countless scientists from other scientific and engineering fields who routinely use products in the form of static and time–variable gravitational fields for their purposes. These arguments naturally motivate the education of subsequent generations of students and researchers in the area of gravitational field, from data collection to modelling. It is essential to provide them with information resources, for example, in the form of a suitable textbook.

The English–language monograph by Heiskanen and Moritz (1967), or its revised edition by Hofmann–Wellenhof and Moritz (2006), are considered the standard textbooks on physical geodesy, aimed primarily at university students. A collection of advanced topics in physical geodesy, suitable for doctoral students and researchers, was published in (Moritz, 1989). Guo (2023) wrote what is currently the most up–to–date textbook on the theoretical foundations of physical geodesy in English. In English–language literature, there is a multitude of geoscience and multidisciplinary titles, particularly monographs and handbooks, that dedicate specific chapters to the gravitational field; see, for example, (Torge and Müller, 2012; Sansò and Sideris, 2013; Freeden et al., 2015; Michel, 2022). Several unique publications on physical geodesy have also emerged from the Czech–Slovak region. Notable examples include the classical textbooks (Pick et al., 1973; Burša, 1977; Burša

and Pěč, 1988) and university lecture notes (Mojzeš, 1989; Janák et al., 2006; Zeman, 2008; Janák, 2010; Zeman, 2010).

When it comes to teaching physical geodesy, there is clearly a wealth of material to choose from. However, it must be recognised that these sources may not reflect the most modern theoretical and technological advancements. Some sources even visibly follow the exposition of the classical monograph (Heiskanen and Moritz, 1967), or devote extensive coverage to topics that are currently outdated or even irrelevant. These reasons, combined with the results of my own systematic scientific effort, have contributed to the creation of the new lecture notes you are currently reading.

The textbook presented here consists of twelve chapters, which can be divided into three parts. The introductory part in Chap. 1 introduces the concept of "geodesy" as it is understood by the International Association of Geodesy. Chapters 2–4 focus sequentially on coordinate and reference systems; fields, differential operators, and tensors; and orthogonal systems of functions. Their purpose is to present the basic notation and nomenclature, certain fundamental concepts, and mathematical functions, and they should be understood as the second preparatory part. Specific problems of physical geodesy are not addressed until the third part, in Chaps. 5–12. In these, we systematically cover the solution of the Laplace equation, selected topics from potential theory, the expansion of the gravitational potential and its higher–order derivatives into series of harmonic functions, the actual gravitational field and its geometry, the normal and disturbing gravitational fields, gravimetry and heights, external boundary value problems, and practical aspects of integral transformations. A similar structure consisting of motivation, nomenclature, and problem solving, is inherent to my way of processing and delivering information. I routinely use it when writing academic papers, and it seems to be digestible for the reader.

The selected areas of physical geodesy may also be the subject of other textbooks mentioned above. However, these lecture notes are unique, and not only within the Czech–Slovak region, for at least two reasons. Firstly, a wide range of gravitational (gravity) field quantities is considered, in the form of the potential, the three components of the potential gradient, and the six components of the second–rank tensor. Secondly, the mathematical framework presented is not limited to symmetric spherical geometry, but is also systematically developed for the more complex geometry of an oblate ellipsoid.

The actual content and method of exposition may be a subject for extensive discussion. These aspects were not chosen randomly without consideration. On the contrary, they are the result of a longer–term process. The first draft of this textbook was created in 2020 for the purposes of teaching the course "Mathematical and Physical Geodesy" (currently "Physical Geodesy I") at the University of West Bohemia in Pilsen for postgraduate students, and it has crystallised into its present form iteratively. I have also been gradually convinced of the validity of the chosen content and method of exposition by other facts.

Over the course of the years 2022–2025, after completing the courses "Mathematical and Physical Geodesy" or "Physical Geodesy I", several gifted students (amongst others) were involved in basic research projects in the field of gravitational field modelling. All of these students were capable of solving complex tasks and actively presented the results of their research activities at scientific symposia, in professional publications, or as part of their final theses. I dare say, therefore, that the presented textbook truly has the potential to provide a solid foundation in the subject of physical geodesy in direct connection with current research problems and challenges.

In Pilsen, Czech Republic, April 2026 Michal Šprlák

Preface to the English version

Originally, I wrote these lecture notes in Czech for postgraduate students at the University of West Bohemia in Pilsen. Right after publication in May 2026, and even prior to that, I had no intention of translating them into a more universal language, such as English. Perhaps the main reason for this narrow focus was my heavy workload at the time.

In September 2026, I attended the XI Hotine–Marussi Symposium in Genoa, Italy. During a coffee break, I spoke with my dear international colleague Dr. Ropesh Goyal from the Indian Institute of Technology in Varanasi, Uttar Pradesh. He had noticed my posts promoting the Czech lecture notes on social media, ordered a hard copy of the publication, and attempted to translate the Czech version into a language he could understand. He eventually gave up on page 8. At the coffee break, I appreciated his order and admired his translation effort, but I was still not fully convinced of the value of an English version.

As is my routine after every scientific event, I reflected on the XI Hotine–Marussi Symposium on my way home. My conversation with Dr. Goyal had clearly left an impression on me. Most importantly, I realised that international peers were genuinely interested in my work and might welcome alternative resources in physical geodesy. At that moment, I changed my perspective and decided to share these ideas internationally.

Initially, I thought the translation would be fast, given the modern translation tools at our disposal. Believe it or not, the raw output sounded like the tools themselves – artificial – and served only as a rough draft. Consequently, I spent two additional rounds reading and rephrasing the text to deliver a human, natural, and faithful English adaptation of the original Czech lecture notes.

International readers should keep three aspects in mind. First, the lecture notes are specifically tailored for postgraduate students in the Geomatics study program at the Faculty of Applied Sciences, University of West Bohemia in Pilsen. Second, the notes frequently cite Czech and Slovak sources that are readily accessible and clear to our local students; readers can substitute respective publications in their own languages as needed. Third, the depth of coverage varies across topics. This variation was intentional, as more general expositions were often explored in detail during practical sessions and exercises

whose context is known primarily to our own students.

Whatever the content, structure, explanatory approach, or level of detail, one can never please everyone. Nevertheless, I hope that readers find this publication – or parts of it – useful.

In Pilsen, Czech Republic, November 2026 Michal Šprlák

Acknowledgements

I would like to thank everyone who participated directly or indirectly in this work. In particular, I would like to express my gratitude to all my mentors, colleagues, and students for countless professional discussions and suggestions. Prof. Jan Kostelecký was kind enough to review the Czech version of the manuscript. I sincerely thank him for the time spent reading the text and for preparing the peer review report. Also, I would like to thank Mr. Jakub Pokorný at the Publishing House of the University of West Bohemia in Pilsen for all his editorial work on the publication of these lecture notes. Last but not least, I am grateful to my family for their constant moral support.

List of abbreviations

BIH Bureau International de l'Heure

CHAMP Challenging Minisatellite Payload

CSR Center for Space Research

DORIS Doppler Orbitography and Radiopositioning Integrated by Satellite

ESA European Space Agency

ETRF European Terrestrial Reference Frame

ETRS89 European Terrestrial Reference System 89

EUREF Regional Reference Frame Sub–Commission for Europe

GFZ Helmholtz–Zentrum für Geoforschung

GGOS Global Geodetic Observing System

GLONASS Globalnaja navigacionnaja sputnikovaja sistěma

GNSS Global Navigation Satellite System

GOCE Gravity Field and Steady–State Ocean Circulation Explorer

GPS Global Positioning System

GRACE Gravity Recovery and Climate Experiment

GRACE–FO GRACE Follow–On

GRAIL Gravity Recovery And Interior Laboratory

GRS Geodetic Reference System

IAG International Association of Geodesy

ICGEM International Centre for Global Earth Models

ICRF International Celestial Reference Frame

ICRS International Celestial Reference System

IERS International Earth Rotation and Reference Systems Service

ISC International Science Council

ITRF International Terrestrial Reference Frame

ITRS International Terrestrial Reference System

IUGG International Union of Geodesy and Geophysics

LLR Lunar Laser Ranging

NASA National Aeronautics and Space Administration

NAVSAT Navy Navigation Satellite System

RCR Remove–Compute–Restore

SLR Satellite Laser Ranging

VLBI Very Long Baseline Interferometry

WGS World Geodetic System

List of Figures

2.1 Cartesian coordinate system in 3D 15
2.2 Global and local spherical coordinate systems 16
2.3 Polar spherical coordinate system 18
2.4 Global and local Gauss ellipsoidal coordinate systems 20
2.5 Global and local Jacobi ellipsoidal coordinate systems 23

6.1 Geometry of the scalar Newton law of gravitation 64
6.2 Geometry of the vector Newton law of gravitation 64
6.3 Gravitational attraction of a realistic body 65
6.4 Gravitational attraction of a point mass in the Cartesian coordinate system in 3D 66
6.5 Directional derivative of the gravitational potential 68
6.6 Point mass m and computation point P in the Cartesian coordinate system in 3D. 71
6.7 Surface S and mass with thickness ζ 74

7.1 General body and spherical geometry 78
7.2 Infinitely thin layer and spherical geometry 84
7.3 General body and ellipsoidal geometry 88
7.4 Infinitely thin layer and ellipsoidal geometry 94

8.1 Gravitational, centrifugal, and gravity force 98
8.2 Equipotential surfaces and plumb lines 101
8.3 A curve in the planar reference system 103
8.4 Equipotential surface and plumb line at point P 104
8.5 Natural reference frame 108

9.1 Equipotential ellipsoid 110
9.2 Definition of geoid height 119
9.3 Definition of height anomaly 120
9.4 Gravity acceleration vectors at point P 120
9.5 Gravity acceleration vectors at points P_0 and Q_0 121
9.6 Gravity acceleration vectors at points P and Q 122

9.7 Deflection of the vertical 122
9.8 Relationship between the geoid and the deflection of the vertical . . . 125

10.1 Asymmetrical free fall 130
10.2 Symmetrical free fall 131
10.3 Absolute gravimeter FG5 133
10.4 Absolute gravimeter A10 (Source: `https://scintrexltd.com/`) . . . 134
10.5 Translational principle 135
10.6 Rotational principle 136
10.7 Lever spring balance 136
10.8 Relative gravimeter Scintrex CG–5 137
10.9 Relative gravimeter Burris ZLS (Source: `https://www.applied-gravity.com/gravimeter.html`) 138
10.10 Schematic diagram of a superconducting gravimeter 138
10.11 Superconducting gravimeter in a laboratory (Source: `https://www.ecgs.lu/wulg/gravimeters/`) 139
10.12 Schematic diagram of a torsion balance 140
10.13 Torsion balance (Pekár, 1928) 140
10.14 The CHAMP satellite mission (Source: `https://www.gfz.de/`) 142
10.15 The GRACE satellite mission (Source: `https://grace.jpl.nasa.gov/`) 144
10.16 The GOCE satellite mission (Source: `https://www.esa.int/`) 146
10.17 The concept of physical heights 148

11.1 General body and spherical geometry 155
11.2 General body and ellipsoidal geometry 168

12.1 Nomenclature on a sphere and its division into near and far zones . . 180
12.2 Behaviour of the integral kernel according to Eq. (11.55) 183
12.3 Behaviour of the truncation error coefficients Q_n 183
12.4 Discretisation of the near zone 186
12.5 Behaviour of the truncation error coefficients Q_n and $Q_n^{(M)}$ 189

List of Tables

3.1 Overview of elementary quantities, differential operators, and the second–rank tensor in spherical coordinates. 38
3.2 Overview of elementary quantities, differential operators, and the second–rank tensor in Jacobi ellipsoidal coordinates. 42

Contents

Preface to the Czech version i

Preface to the English version iv

Acknowledgements vi

List of abbreviations vii

List of figures ix

List of tables xi

1 Geodesy – definition, scope, history, international organisations, institutions, and literature 1

1.1 Definition and scope of geodesy 1
1.2 History of geodesy 2
1.2.1 The Earth as a sphere 3
1.2.2 The Earth as an ellipsoid of revolution 4
1.2.3 The Earth as a geoid and international cooperation 5
1.2.4 Geodesy in 3D 6
1.2.5 Geodesy in 4D 7
1.3 International scientific organisations 8
1.3.1 International Science Council (ISC) 8
1.3.2 International Union of Geodesy and Geophysics (IUGG) 8
1.3.3 International Association of Geodesy (IAG) 9
1.4 National geodetic institutions 11
1.5 Geodetic literature 12

2 Coordinate and reference systems 14

2.1 Basic coordinate systems 15
2.1.1 Cartesian coordinate system in 3D 15
2.1.2 Global spherical coordinate system 16
2.1.3 Local spherical coordinate system 17

2.1.4 Polar spherical coordinate system 18
2.1.5 Global Gauss ellipsoidal coordinate system 19
2.1.6 Local Gauss ellipsoidal coordinate system 21
2.1.7 Global Jacobi ellipsoidal coordinate system 22
2.1.8 Local Jacobi ellipsoidal coordinate system 24
2.2 Reference systems used in geodesy 25
2.2.1 International Terrestrial Reference System (ITRS) 25
2.2.2 International Celestial Reference System (ICRS) 26
2.2.3 World Geodetic System (WGS) 26
2.2.4 European Terrestrial Reference System 89 (ETRS89) 27
2.2.5 Geodetic Reference System (GRS) 27

3 Fields, differential operators, and tensors 28
3.1 Definition of a field 28
3.2 Differential operators and tensors in Cartesian coordinate system in 3D 29
3.2.1 Hamilton (del, nabla) operator 29
3.2.2 Gradient 29
3.2.3 Divergence 30
3.2.4 Rotation 31
3.2.5 Laplace operator 31
3.2.6 Tensors 32
3.3 A general form of differential operators and second–rank tensors 33
3.4 Expression of elementary quantities, differential operators, and the second–rank tensor in spherical coordinates 35
3.5 Expression of elementary quantities, differential operators, and the second–rank tensor in Jacobi ellipsoidal coordinates 38
3.6 Gradient and the components of the second–rank tensor in the polar spherical coordinate system 42

4 Orthogonal systems used in planetary gravitational field modelling 45
4.1 Legendre polynomials 45
4.1.1 Legendre polynomials of the first kind and their properties . . . 45
4.1.2 Legendre polynomials of the second kind 47
4.2 Associated Legendre functions 48
4.2.1 Associated Legendre functions of the first kind and their properties 48
4.2.2 Normalised Legendre polynomials and normalised associated Legendre functions of the first kind 49
4.2.3 Associated Legendre functions of the second kind 50

4.3 Surface spherical harmonic functions 50
4.3.1 Surface spherical harmonic functions and their properties 50
4.3.2 Normalised surface spherical harmonic functions 52
4.4 Extension for the surface of an ellipsoid of revolution 53

5 Solution of the Laplace equation **55**
5.1 Solution of the Laplace equation in spherical coordinates 56
5.2 Solution of the Laplace equation in Jacobi ellipsoidal coordinates 59

6 Selected topics from the potential theory **64**
6.1 Basic concepts 64
6.2 Description of the gravitational field by the gravitational potential . . . 67
6.3 Properties of the gravitational potential of a point mass 69
6.4 Gravitational potential of a general body and its properties 72
6.5 The gravitational potential of an infinitely thin layer and its properties 74

7 Expansion of the gravitational potential and its derivatives into a series of harmonic functions **77**
7.1 Expansion into a series of spherical harmonic functions 78
7.1.1 Gravitational potential of a general body 78
7.1.2 Gravitational potential of a general body above the minimum Brillouin sphere 81
7.1.3 Gravitational potential of an infinitely thin layer above the minimum Brillouin sphere 84
7.1.4 Determination of surface density from the gravitational potential above the minimum Brillouin sphere 86
7.2 Expansion into a series of ellipsoidal harmonic functions 87
7.2.1 Gravitational potential of a general body 87
7.2.2 Gravitational potential of a general body above the minimum Brillouin ellipsoid of revolution 91
7.2.3 Gravitational potential of an infinitely thin layer above the minimum Brillouin ellipsoid of revolution 94
7.2.4 Determination of surface density from the gravitational potential above the minimum Brillouin ellipsoid of revolution 96

8 The actual gravity field and its geometry **98**
8.1 The actual gravity field 98
8.2 Equipotential (level) surfaces and plumb lines 101
8.3 Curvature of equipotential surfaces and plumb lines 103

8.3.1 Curvature of a curve . . . 103
8.3.2 Curvature of an equipotential surface . . . 104
8.3.3 Curvature of a plumb line . . . 106
8.4 The natural reference system . . . 107

9 The normal and disturbing gravity fields 109
9.1 The normal gravity field . . . 109
9.1.1 The normal gravity field of an equipotential ellipsoid . . . 109
9.1.2 Geodetic Reference System 1980 . . . 117
9.2 Disturbing gravity field . . . 118
9.2.1 Quantities of the disturbing gravity field . . . 118
9.2.2 Mathematical relationships between the disturbing potential and other quantities of the disturbing gravity field . . . 123

10 Gravimetry and heights used in geodesy 128
10.1 Gravimetry . . . 128
10.1.1 Terrestrial gravimetry . . . 129
10.1.2 Satellite gravimetry . . . 141
10.1.3 Gravimetric reference system . . . 147
10.2 On heights used in geodesy . . . 147
10.2.1 The concept of physical heights and geopotential numbers . . . 148
10.2.2 Dynamic heights . . . 149
10.2.3 Orthometric heights . . . 150
10.2.4 Normal orthometric heights . . . 151
10.2.5 Normal heights . . . 151
10.2.6 Ellipsoidal heights . . . 152
10.2.7 Vertical reference system and frame . . . 153

11 External boundary value problems of potential theory and other integral transformations 154
11.1 External boundary value problems and derived integral transformations in the spherical approximation . . . 155
11.1.1 Solution to the external spherical Dirichlet boundary value problem 156
11.1.2 Solution to the external spherical Neumann boundary value problem 158
11.1.3 Solution to the external spherical Stokes boundary value problem 160
11.1.4 Derived spherical integral transformations . . . 162
11.1.5 Closed forms of isotropic integral kernels . . . 165
11.2 External boundary value problems and derived integral transformations in the ellipsoidal approximation . . . 167

11.2.1 Solution to the external ellipsoidal Dirichlet boundary value problem . . . 169
11.2.2 Solution to the external ellipsoidal Neumann boundary value problem . . . 171
11.2.3 Derived integral transformations . . . 173

12 Practical aspects of the numerical computation of disturbing gravity field quantities by integral transformations 178
12.1 Decomposition of global integration . . . 179
12.1.1 Decomposition of global integration in the spatial domain . . . 179
12.1.2 The Remove–Compute–Restore method . . . 184
12.2 Discretisation of the integral for computing the near–zone effect . . . 186
12.3 Modification of the integral kernel . . . 187
12.4 The inverse problem . . . 190

References 192

Chapter 1

Geodesy – definition, scope, history, international organisations, institutions, and literature

1.1 Definition and scope of geodesy

Geodesy is one of the oldest scientific disciplines, along with astronomy, physics, geography, and mathematics. Its name comes from the two Greek words $\gamma\eta$ (meaning "Earth") and $\delta\alpha\iota\omega$ (meaning "I divide"), which literally translates to "the division of the Earth."

From a historical perspective, the scope of geodesy and its definition have been gradually modified. A classical definition was provided by the German geodesist Friedrich Robert Helmert (1843–1917) as follows (Helmert, 1880): "Geodesy is the science of measuring and mapping the Earth's surface." Although this definition of geodesy can still be found in textbooks, it does not reflect the current scope of the field. The Czech Terminological Dictionary of Surveying and Real Estate Cadastre (`https://www.slovnikcuzk.eu`) provides a more comprehensive and newer definition: "Geodesy is a natural science, one of the Earth sciences, which uses geometric and physical methods to obtain data of a metric and physical nature about the Earth; it is simultaneously a technical discipline that provides geometric data for map production and for the needs of other disciplines." According to the definition by the Czech Office for Surveying, Mapping and Cadastre, geodesy is therefore both a natural science and a technical discipline. In the terminology of other countries, two distinct terms are usually used to denote the natural science (e.g., geodesy in English, geodäsie in German) and the technical discipline (e.g., surveying in English, vermessungswesen in German).

The Czech Terminological Dictionary of Surveying and Real Estate Cadastre also contains a definition of surveying as: "A summary of geodetic, photogrammetric, and carto-

graphic tasks, including technical activities in the real estate cadastre. Particularly in the names of institutions, it often replaces the terms geodesy and cartography." In the subsequent exposition, we will proceed in contradiction to the Czech Terminological Dictionary of Surveying and Real Estate Cadastre, yet completely logically and in accordance with the practices in other countries. Specifically, we will use the term geodesy for the natural science and the term surveying for the technical discipline.

For the natural science specified in this way, we will now present what is probably the most up–to–date and general definition: "Geodesy is the scientific discipline concerned with the measurement and representation of the shapes and gravity fields of planetary bodies in three–dimensional space, along with their changes over time." It is important to emphasise that geodesy has a unique object of study, its own terminology, and its own scientific methods. It therefore meets the requirements of a science discipline.

In the literature, we can find a wide range of modifiers specifying a particular branch of geodesy, for example, physical geodesy, satellite geodesy, mathematical geodesy, engineering geodesy, planar geodesy, higher geodesy, tensor geodesy, polar geodesy, differential geodesy, inertial geodesy, etc.. The use of these modifiers is quite subjective and does not alter the scope (the basic functions) of geodesy, which are:

- positioning,
- the gravitational field,
- temporal variations (of both position and the gravitational field).

In the following chapters, we will focus primarily on the gravitational field, the detailed study of which is precisely the domain of physical geodesy.

1.2 History of geodesy

The historical roots of geodesy (as well as of surveying) date back to the times of the first settlements and tribes, tens of thousands of years ago. Early humans faced the task of spatial localisation during the daily search for food, the selection of a suitable camp–site, or seasonal migration.

From the times of the ancient civilisations of Egypt, India, China, and Mesopotamia, few direct mentions of geodesy exist. The rich heritage of these first modern civilisations, such as monumental buildings and calendars, may be at the very least a sign of awareness on the Earth's orientation in space. The construction of irrigation systems and the regulation of watercourses through levelling provide evidence that the gravitational field, which directs how water actually flows, was respected.

In ancient Greece, several prominent scholars systematically engaged in determining the shape and size of the Earth. It is with this period that we shall begin our historical account of geodesy. Extensive material on the history of geodesy can be found, for example, in (Todhunter, 1873; Fischer, 1975; Smith, 1986; Torge and Müller, 2012).

1.2.1 The Earth as a sphere

In the past, various views on the shape of the Earth prevailed. Thales (∼625 – ∼547 BC) imagined the Earth as a disc floating in an infinite ocean. Anaximander (∼611 – ∼545 BC) defined the shape of our planet as a cylinder. Pythagoras (∼580 – ∼500 BC) and his school proposed a spherical shape for the Earth. By the time of Aristotle (384–322 BC), this approximation was generally accepted and even justified by observation. Observers noticed the circular shadow of the Earth during lunar eclipses and the apparent rising of an approaching ship on the horizon. In China, the spherical shape of the Earth was recognised later, in the first century AD.

Eratosthenes (276–195 BC) was the first to determine the radius of a spherical Earth, and he is often considered the founder of geodesy. For this purpose, he needed to know the angular deviation between two points on the same meridian and the distance between them. At the time of the summer solstice, he measured (using the sun's rays) an angular deviation of 7° 12′ between the cities of Syene (now Aswan) and Alexandria. He estimated the distance between these cities, likely from Egyptian cadastral maps, to be 5000 stadia (a unit of length equal to about 158.7 m according to the Egyptian standard). The resulting radius of the Earth could therefore have been 6270 km.

Posidonius (135–51 BC) also determined the radius of the Earth. His estimate was a few dozen kilometres smaller than that of Eratosthenes. To achieve this, he used the knowledge of the approximate distance between Alexandria and the island of Rhodes. He estimated the angular separation of these points by observing the star Canopus (while Canopus was on the horizon in Rhodes, the star was at an altitude of 7° 30′ above the horizon in Alexandria).

Claudius Ptolemy (∼75 – ∼151) introduced the geocentric world system in his publications on astronomy (known by its Arabic name Almagest) and geography (Geographike hyphegesis). These works included star catalogues, maps, and lists of geographical coordinates for many locations.

During the Middle Ages, geodesy (as well as other sciences) did not develop significantly in Europe due to the prevailing power of the Church. However, further experiments to determine the radius of the Earth were also carried out outside the European continent. Notable examples include the astronomical and geodetic measurements in China carried out by Nangong Yue and Yi Xing (around 725), and measurements between Baghdad

and Al–Raqqah under the leadership of Caliph Al–Ma'mun (around 827). Later, during the Renaissance, Jean Fernel attempted to determine the Earth's radius around 1525 by taking measurements between Paris and Amiens.

Subsequent advancements in geodesy were fundamentally influenced by the invention of the telescope in the Netherlands (around 1600). Optical devices for measuring directions (angles) began to develop, and the method of triangulation was increasingly applied in surveying practice. This method was used for more than 350 years until the advent of electronic distance meters and artificial satellites. The first triangulation measurements were carried out by Gemma Frisius (1508–1555) in the Netherlands and Tycho Brahe (1546–1601) in Denmark. Snellius (1580–1626) utilised measurements within a triangulation network (between 1614 and 1615) between Bergen op Zoom and Alkmaar to determine the Earth's radius. Between 1669 and 1670, Jean Picard (1620–1682) determined the length of a meridian arc by measuring a triangulation network between Malvoisine and Amiens, establishing the corresponding radius of the Earth as 6328.9 km. This value for the Earth's radius was used by Isaac Newton (1642–1727) himself when formulating his law of universal gravitation.

1.2.2 The Earth as an ellipsoid of revolution

New astronomical observations and revolutionary ideas in physics during the sixteenth and seventeenth centuries had a fundamental influence on the perception of the Earth's shape and its position in the universe. Nicolaus Copernicus (1473–1543) proposed a heliocentric system of the universe, which had already been suggested by Aristarchus (∼310 – ∼250 BC). Johannes Kepler (1571–1630) formulated the laws of planetary motion. Galileo Galilei (1564–1642) laid the foundations of mechanics through the experimental observation of falling bodies.

The flattening of Jupiter at its poles was observed in 1666 by Giovanni Domenico Cassini (1625–1712). Jean Richer (1630–1696) conducted pendulum experiments in Paris and in Cayenne, French Guiana, which is located almost on the equator. In Cayenne, the length of the pendulum had to be shortened for it to oscillate with the same period as in Paris. From this observation, and based on the law of pendulum motion, an increase in gravity towards the poles can be deduced. A similar effect was also confirmed by Edmond Halley (1656–1742) when comparing pendulum measurements on the island of Saint Helena and in London.

Based on these observations and his theoretical work on gravitation and hydrostatics, Isaac Newton proposed an ellipsoid of revolution (flattened at the poles) as an adequate model of the Earth. He also postulated an increase in gravity acceleration from the equator towards the poles. A similar model of the Earth was also proposed by Christiaan Huy-

gens (1629–1695). Jacques Cassini (1677–1756) analysed earlier arc measurements and concluded that the adequate shape of the Earth was an ellipsoid elongated at the poles (a prolate ellipsoid).

The French Academy of Sciences decided to resolve the contradiction between the followers of Newton and Cassini by organising two unique campaigns to determine the length of meridian arcs in the equatorial and polar regions. Pierre Louis Maupertuis (1698–1759) and Alexis Claude Clairaut (1713–1765) took part in an expedition to Lapland between 1736 and 1737. Pierre Bouguer (1698–1758), Charles Marie de La Condamine (1701–1774), and Louis Godin (1704–1760) participated in the second expedition to the territory of present–day Ecuador between 1735 and 1744 (Ferreiro, 2013). Both expeditions experimentally confirmed the flattening of the Earth at the poles, thereby validating Newton's hypothesis regarding the shape of our planet.

In 1743, Clairaut found a mathematical relationship between flattening and gravity. Pierre–Simon de Laplace (1749–1827) utilised this theorem to determine the Earth's flattening from just fifteen gravity measurements. A significantly more accurate determination of the Earth's flattening became possible only with the advent of satellite methods.

Following the acceptance of the ellipsoid of revolution as a sufficiently accurate representation of the Earth's shape, a whole series of arc measurements was carried out. From the length of the meridian arc, the natural value of one conventional metre was also derived in 1799. This unit was finally accepted in 1875 by an international agreement at a conference in Paris.

The theoretical foundations for the gravity field determination were also formulated during the same time period. Among the contributors were the prominent figures in world science, such as Jean le Rond d'Alembert (1717–1783), Joseph–Louis Lagrange (1736–1813), Pierre–Simon de Laplace, Adrien–Marie Legendre (1752–1833), and later also Carl Friedrich Gauss (1777–1855) and George Green (1793–1841). George Gabriel Stokes (1819–1903) derived the integral that transforms gravity measurements into geoid undulations.

1.2.3 The Earth as a geoid and international cooperation

At the beginning of the nineteenth century (in 1806), Adrien–Marie Legendre and Carl Friedrich Gauss developed the method of least squares and, among other applications, used it in the adjustment of triangulation networks. In their adjustments, as well as subsequent ones, the influence of the angular deviation between the actual plumb line and the ellipsoidal normal became noticeable. The effect of the vertical deflection led to a more precise definition of the Earth's shape. Carl Friedrich Gauss and Friedrich Wilhelm Bessel (1784–1846) strictly distinguished between the physical surface of the

Earth, the geoid as a mathematical surface, and the ellipsoid of revolution as a surface approximating the geoid. Friedrich Robert Helmert systematically summarised all findings regarding the shape of the Earth and its approximations (Helmert, 1880). Despite this progress, the method of triangulation had been preferred in determining the dimensions of local ellipsoids of revolution and in establishing national triangulation networks until the satellite era. In addition to measurements of a purely geometric nature, regular gravity measurements using pendulums also began to be carried out.

The extent of triangulation networks expanded significantly. Historically notable examples included the measurements between the cities of Göttingen and Altona (1821–1824); the triangulation across Great Britain, France, Spain, and Algeria along the Paris Meridian; and the so–called Struve Geodetic Arc (1816–1855), starting in northern Norway and ending on the coast of the Black Sea. The interconnection of national triangulations initiated international cooperation and ultimately led to the founding of the International Association of Geodesy, see Subsect. 1.3.3.

From approximately 1880, national vertical height systems were established, though independently from horizontal networks. Expensive and time–consuming levelling measurements were carried out relative to a chosen mean sea level, the so–called datum point of the levelling network.

In addition to an international standard of length, it was necessary to introduce a world time system and a prime meridian to define the zero geographical longitude. At an international conference held in Washington, D.C. in 1884, the Greenwich Meridian was adopted as the baseline, and the mean solar day was selected as the unit of time.

1.2.4 Geodesy in 3D

The core idea of three–dimensional geodesy lies in the joint adjustment of angular, distance, and levelling measurements within a single mathematical model. This yields the 3D coordinates of points. This approach was proposed by Ernst Heinrich Bruns (1848–1919). However, its practical implementation was significantly limited due to the independence of vertical and horizontal networks. Antonio Marussi (1908–1984) and Martin Hotine (1898–1968) later revived this approach, whilst also incorporating the geometry of the gravity field. Mikhail Sergeevich Molodenskii (1909–1991) demonstrated that the physical surface of the Earth and its external gravity field can be determined from surface measurements, without any reliance on the geoid.

The first electronic distance meters came into use in the 1950s. They brought a revolutionary automation of surveying procedures, particularly in land surveying. In 1957, the first artificial Earth satellite, Sputnik 1, was launched. The Czechoslovak astronomer and geodesist Emil Buchar (1901–1979) estimated the Earth's flattening from visual observa-

tions of the Sputnik 1 and 2 satellites. Artificial satellites orbiting our planet initiated the era of realising global geocentric 3D reference systems.

Primarily for military purposes, comprehensive satellite positioning systems were established in the 1980s, namely GPS (Global Positioning System) and GLONASS (Globalnaja navigacionnaja sputnikovaja sistěma). The set of existing positioning systems is today known by the acronym GNSS (Global Navigation Satellite System). Alongside GNSS, other space–geodetic techniques evolved based on technological advancements, known as DORIS (Doppler Orbitography and Radiopositioning Integrated by Satellite), LLR (Lunar Laser Ranging), SLR (Satellite Laser Ranging), and VLBI (Very Long Baseline Interferometry). The transformation between modern reference frames and classical frames has become one of the challenges in geodesy.

The geodetic community has also made significant progress in Earth's gravity field modelling. Research in this area was often theoretical in nature due to the confidentiality or unavailability of gravity measurements. Derived models of the Earth's gravity field were highly inhomogeneous and their application was rather limited. A revolution in this regard arrived after 2000, when four satellite missions were launched in succession, known by the acronyms CHAMP (Challenging Minisatellite Payload, 2000–2010), GRACE (Gravity Recovery and Climate Experiment, 2002–2017), GOCE (Gravity Field and Steady–State Ocean Circulation Explorer, 2009–2013), and GRACE–FO (GRACE Follow–On, 2018–present).

1.2.5 Geodesy in 4D

The origins of four–dimensional geodesy date back to the end of the nineteenth century. Between 1884 and 1885, Karl Friedrich Küstner (1856–1936) explained a change in the geographical latitude of an observed star as polar motion. Ernst von Rebeur–Paschwitz (1861–1895) observed the effect of tides during pendulum gravity measurements between 1889 and 1893. Around 1880, precise levelling and the recording of sea levels using tide gauges commenced in Scandinavia. The objective of these measurements was to determine post–glacial rebound. In Japan and the USA, crustal deformations caused by seismic activity have been monitored for more than 100 years.

At present, variations in Earth's rotation and tectonic plate movements are regularly monitored using space–geodetic observation techniques. Local monitoring networks have been stabilised at tectonic plate boundaries and in the vicinity of volcanoes. Variations in the global gravity field were measured between 2002 and 2017 by the GRACE satellite mission, and they have been monitored by the GRACE–FO mission since 2018. Local variations in the gravity field are detected through continuous measurements from superconducting gravimeters.

Enormous efforts are being made worldwide today to understand the dynamics of the Earth system and global change processes. Geodesy makes an indispensable contribution to this endeavour through its infrastructure, which is known as GGOS (Global Geodetic Observing System).

1.3 International scientific organisations

1.3.1 International Science Council (ISC)

The International Science Council (`https://council.science`) is a non–governmental organisation that serves as an umbrella body for scientific organisations (associations, unions, etc.) in the social and natural sciences. The ISC was founded on 4 July 2018 through the merger of two prior organisations, namely the International Council for Science and the International Social Science Council.

The ISC convenes and mobilises the international scientific community on issues of fundamental scientific and public importance. Its activities focus on:

- supporting international scientific research and promoting science communication relevant to international policy issues,
- supporting the capacity of science to contribute to solving critical problems,
- safeguarding the free and responsible practice of science.

The ISC co–sponsors international research programmes, networks, and committees. At the end of 2026, the ISC had over 250 member organisations. This includes 40 international scientific unions and associations, over 140 national and regional scientific organisations, and more than 20 affiliated members.

1.3.2 International Union of Geodesy and Geophysics (IUGG)

The International Union of Geodesy and Geophysics (`https://iugg.org`), one of the 40 member unions and associations within the ISC, is a non–governmental organisation dedicated to the scientific study of the Earth and its environment using geophysical and geodetic techniques. The IUGG was founded in 1919 in Brussels.

The objectives of the IUGG are the promotion and coordination of studies related to the physical, chemical, and mathematical representation of the Earth. This includes the geometric shape, internal structure, gravity and magnetic fields, seismicity, volcanism, the hydrological cycle, glaciers, oceans, atmosphere, ionosphere, and magnetosphere of the Earth. In addition to the Earth, other planetary bodies are also subjects of interest to the IUGG.

Vážení čtenáři, právě jste dočetli ukázku z knihy ***Physical Geodesy*.**
Pokud se Vám ukázka líbila, na našem webu si můžete zakoupit celou knihu.